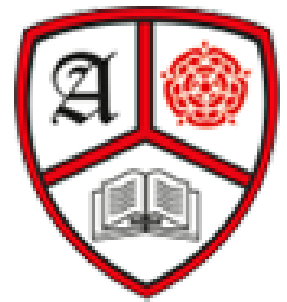


# Anderton Primary School

## Maths Mastery Calculation Policy

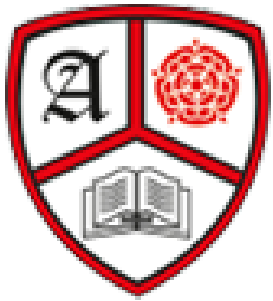


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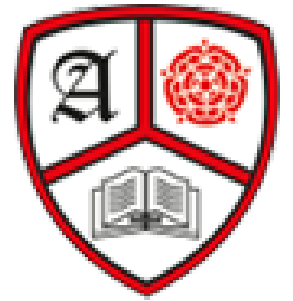
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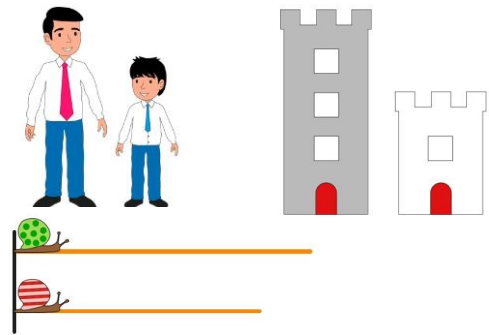
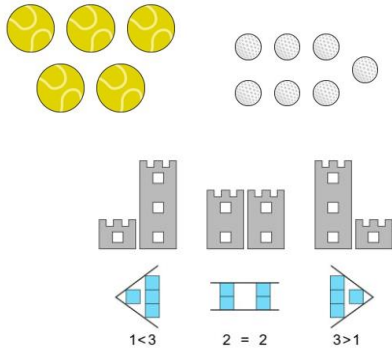
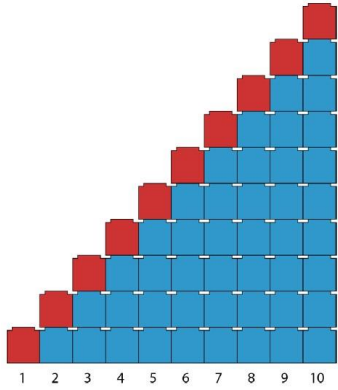
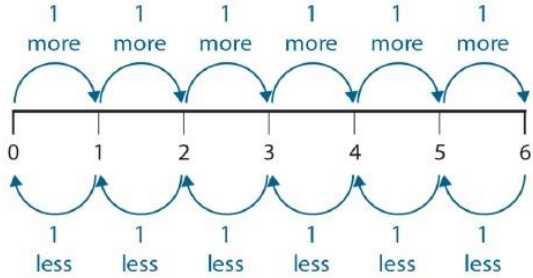
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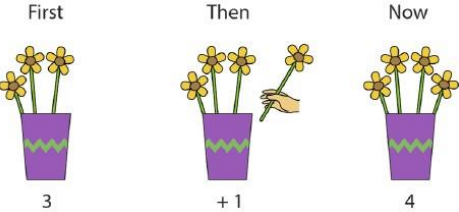
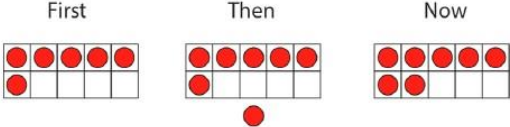
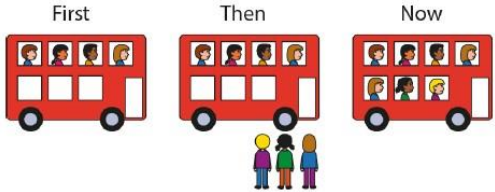
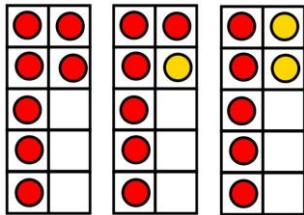
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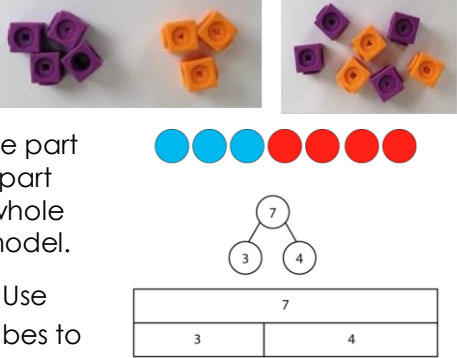
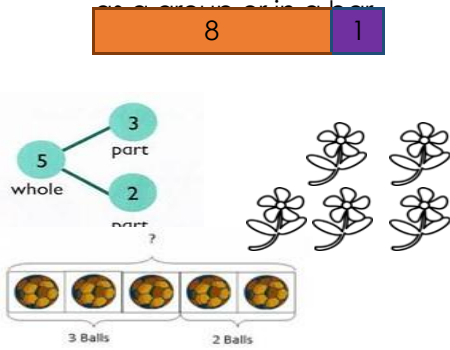

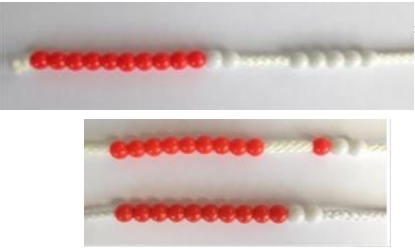
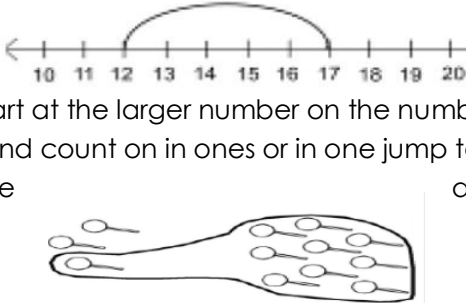
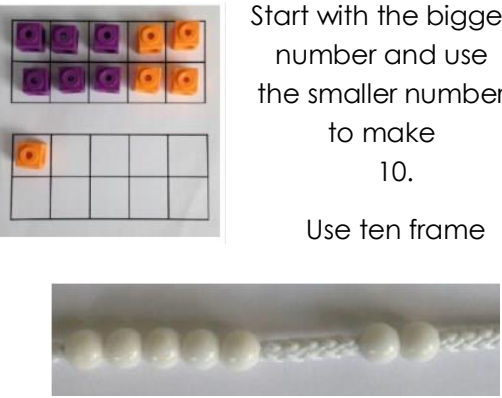
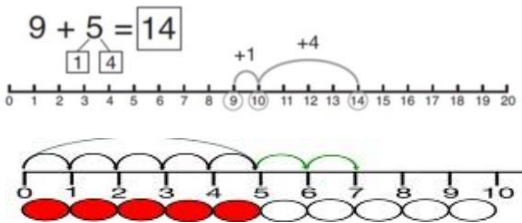


# Year 1 Addition



Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
<p>Comparing Objects, groups of objects</p> <p>Length, weight, mass, heavier, lighter, same, equal</p>	<p>People's height, distance, mass.</p> <p>Use of pan balances using Numicon or similar to show equivalence, &lt; &gt;</p> <p>Comparing multiple objects</p> <p>Use of concrete materials eg. Compare bears, jewels, cubes etc to create groups of different sizes to compare</p>		
<p>Using &lt; &gt; and =</p> <p>Fewer, more, less than, more than, equal to, fewer than</p>	<p>Use a multilink staircase in two colours</p>		<p>Use variation with missing boxes and missing symbols.</p> <p>3 ○ 4      4 &gt; □</p> <p>2 ○ 2      □ &lt; 6</p>
<p>Finding one more, finding one less</p>			<p>One more/less sentences – example one:</p> <p>1 more than 3 is □</p> <p>1 less than 2 is □</p> <p>1 more than □ is 1</p> <p>1 less than □ is 1</p>

Objective, Strategy & Key Vocabulary	Concrete	Pictorial	Abstract
Adding 1 gives 1 more	<p>First                  Then                  Now</p>  <p>3                  + 1                  4</p>	<p>First                  Then                  Now</p> 	$\begin{array}{ccc} 6 & + 1 & 7 \\ \hline & & \rightarrow \end{array}$ $6 + 1 = 7$
Augmentation— increasing an amount	<p>Use FIRST, THEN, NOW and range of practical situations for showing augmentation.</p> <p>E.g. first there were three chn on carpet then 2 more came. Now there are 5 chn on the carpet.</p>	<p>First                  Then                  Now</p> 	$\begin{array}{ccc} 4 & + 3 & 7 \\ \hline & & \rightarrow \end{array}$ $4 + 3 = 7$
Stories of numbers within 10	<p>Children should work with doubled sided counters and ten frame.</p> <p>Start with 7 red, turn one over, tell me the 'story'?</p> <p>Turn one more over. What is the 'story'?</p> <p>Continue.</p> <p>Complete this for stories of all numbers up to 10.</p>	 <p>7 + 0 = 7 6 + 1 = 7 5 + 2 = 7 etc</p> <p>Complete for all numbers up to 10</p>	$\begin{array}{l} 7 + 0 = 7 \\ 6 + 1 = 7 \\ 5 + 2 = 7 \\ 4 + 3 = 7 \\ 3 + 4 = 7 \\ 2 + 5 = 7 \\ 1 + 6 = 7 \\ 0 + 7 = 7 \end{array}$

Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together</p> 	 <p><math>4 + 3 = 7</math></p> <p><math>10 = 6 + 4</math></p> <p>Use the part whole diagram as shown above to move into the abstract.</p>
<p>Regrouping to make 10.</p> <p><i>This is an essential skill for column addition later.</i></p>	 <p><math>= 11</math></p> <p>2 more than 5.</p>	 <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p> <p><math>3 + 9 =</math></p>	<p><math>7 + 4 = 11</math></p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
<p>Represent &amp; use number bonds and related subtraction facts within 20</p>	 <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frame</p>	<p>Use pictures or a number line. Regroup or partition the smaller number using the part whole model to make 10.</p> 	<p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.' '8 is 3 more than 5.'</p>

Adding 1 and 2

Bonds to 10

Adding 10

Bridging/  
compensating

Doubles

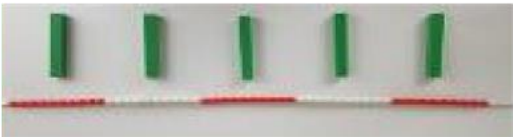
Adding 0

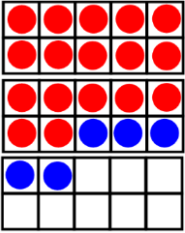
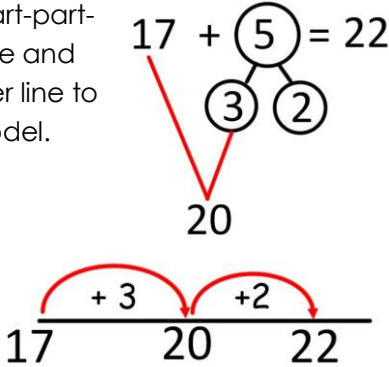
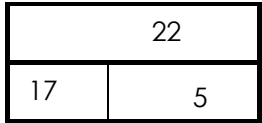

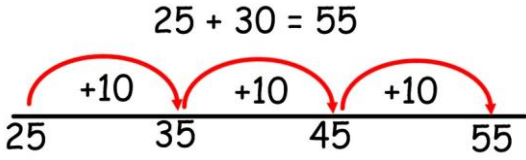
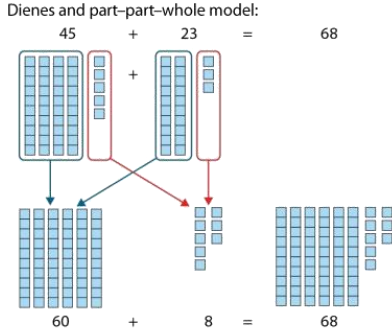
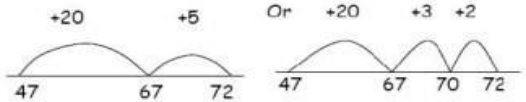
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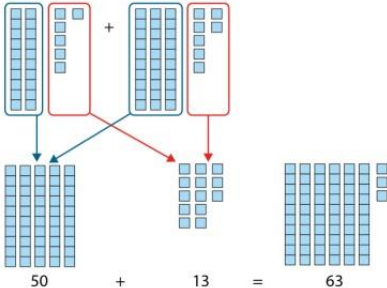
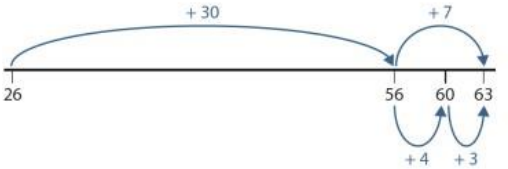


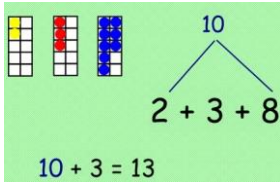
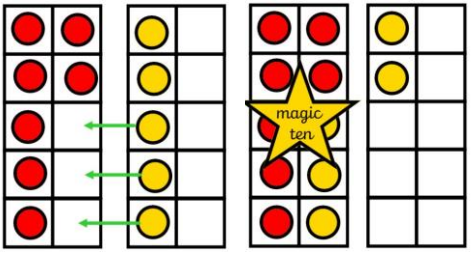
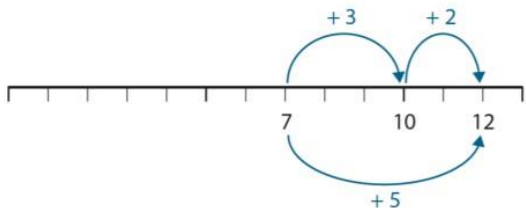
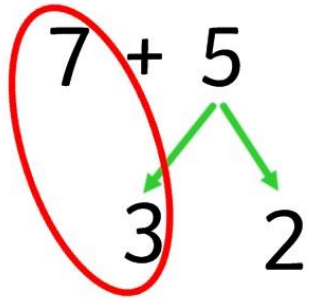
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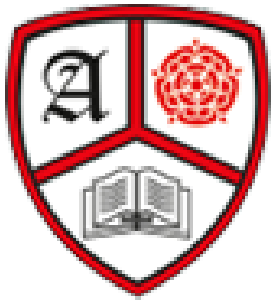
Y2  
facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

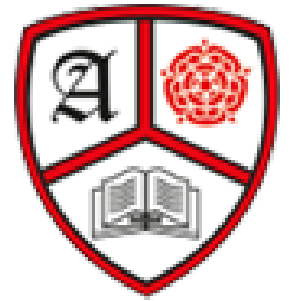
Objective & Strategy & Key Vocabulary	Concrete	Pictorial	Abstract
Adding multiples of ten	<div>50= 30 + 20</div>  <div>Model using dienes and bead strings</div>	<div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div></div></div> <div>__ tens and __ tens makes __ tens</div> <div>Use representations for base ten.</div>	<div>20 + 30 = 50</div> <div>70 = 50 + 20</div> <div>40 + □ = 60</div> <div>□ + 30 = 50</div>
Use known number facts  Part part whole	<div><div>20</div><div><div></div><div></div></div><div><div></div><div></div></div></div> <div>Children explore ways of making numbers within 20</div>	<div><div>20</div><div><div></div><div></div></div></div> <div><div>□ + □ = 20</div><div>20 - □ = □</div><div>□ + □ = 20</div><div>20 - □ = □</div></div>	<div>□ + 1 = 16</div> <div>16 - 1 = □</div> <div>1 + □ = 16</div> <div>16 - □ = 1</div>
Using known facts	<div><div>Ted</div><div><div></div><div></div><div></div></div></div> <div><div>Sam</div><div><div></div><div></div><div></div><div></div><div></div></div></div>	<div><div><div></div><div></div></div><div>+</div><div><div></div><div></div></div><div>=</div><div><div></div><div></div><div></div></div></div> <div><div><div></div><div></div></div><div>+</div><div><div></div><div></div></div><div>=</div><div><div></div><div></div><div></div><div></div></div></div> <div><div><div></div><div></div></div><div>+</div><div><div></div><div></div></div><div>=</div><div><div></div><div></div><div></div><div></div><div></div></div></div> <div>Children draw representations of H,T and O</div>	<div>3 + 4 = 7</div> <div>Leads to</div> <div>30 + 40 = 70</div> <div>Leads to 300 + 400 + 700</div> <div>'3 things and 4 things is always 7 things'</div>
Bar model	<div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div> <div>3 + 4 = 7</div>	<div><div>8</div><div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div></div></div></div> <div>3 + 5 = 8</div>	<div><div>30</div><div><div>14</div><div>16</div></div></div> <div>14 + 16 = 30</div>

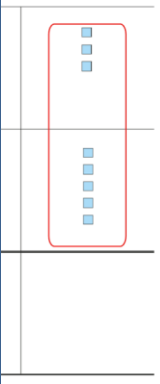
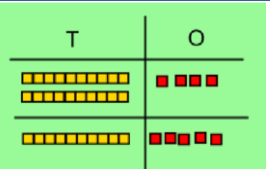

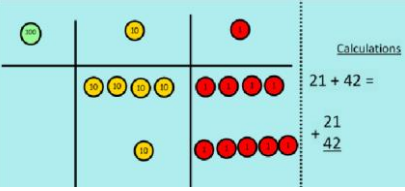
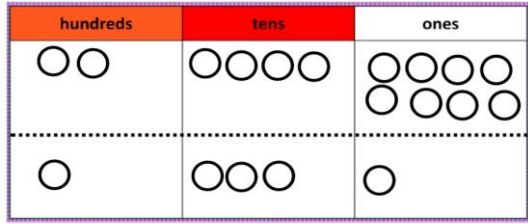
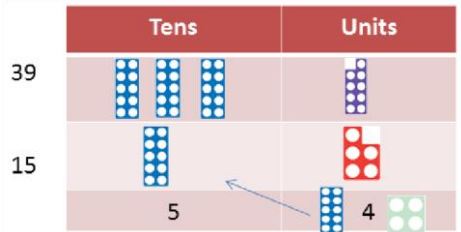
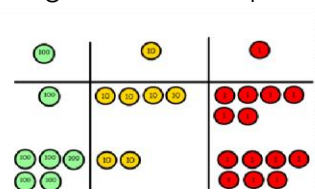
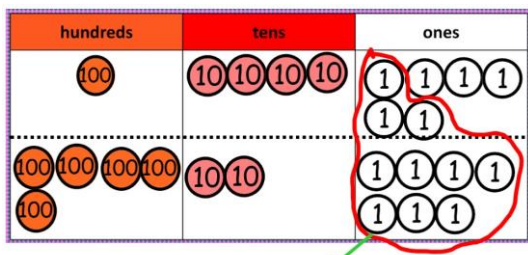
Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
Add a two digit number and ones	 <p> <math>17 + 5 = 22</math>            Use ten frame to make 'magic ten'            Children explore the pattern.  <math>17 + 5 = 22</math>  <math>27 + 5 = 32</math> </p>	<p>Use part-part-whole and number line to model.</p> 	<p> <math>17 + 5 = 22</math>            Explore related facts  <math>17 + 5 = 22</math>  <math>5 + 17 = 22</math>  <math>22 - 17 = 5</math>  <math>22 - 5 = 17</math> </p>  <p> <math>17 + 5</math>  <math>22 = 5 + 17</math>  <math>17 = 22 - 5</math>  <math>5 = 22 - 17</math> </p>
Add a 2 digit number and tens	 <p> <math>25 + 10 = 35</math>            Explore that the ones digit does not change         </p>	 <p> <math>25 + 30 = 55</math> </p>	<p> <math>27 + 10 = 37</math>  <math>27 + 20 = 47</math>  <math>27 + \square = 57</math>  <math>\square + 30 = 67</math> </p>
Add two 2-digit numbers without bridging. 'Friendly numbers'	<p>Model using dienes, place value counters and numicon</p>  <p> <math>45 + 23 = 68</math>  <math>60 + 8 = 68</math> </p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	<p> <math>25 + 47</math>  <math>20 + 5</math>      <math>40 + 7</math>  <math>20 + 40 = 60</math>  <math>5 + 7 = 12</math>  <math>60 + 12 = 72</math> </p>

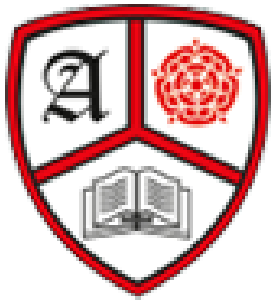
Objective, Strategy Key Vocabulary	Concrete	Pictorial	Abstract
Add any two 2-digit numbers	<p>Dienes and part-part-whole model:</p> $26 + 37 = 63$  $50 + 13 = 63$	$26 + 30 + 7$ 	$24 + 38 = \square$ $38 + 24 = \square$ $29 + \square = 51$ $\square + 22 = 51$
Add three 1-digit numbers	 <p>Combine to make magic 10 first where relevant, or bridge 10 then add third</p>	<p>Use language of fist, then, then, now</p> <p>Pictorial:</p> <p>First      Then      Then      Now</p>  <p>Use part part whole to show magic ten</p>  $2 + 3 + 8$ $10 + 3 = 13$	$4 + 7 + 6 = 10 + 7$ $= 17$ <p>Combine the two numbers that make/ bridge ten then add on the third.</p>
Adding two numbers that bridge 10.	 <p>Use double sided counters and ten frames. Move counters to fill the ten frame and make Magic 10</p>	 <p>Show on a number line how 5 is portioned into adding three, then adding 2.</p>	



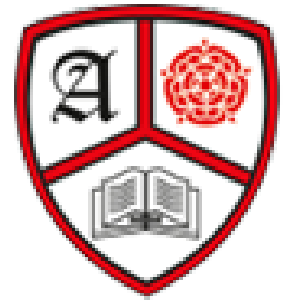
# Year 2 Addition

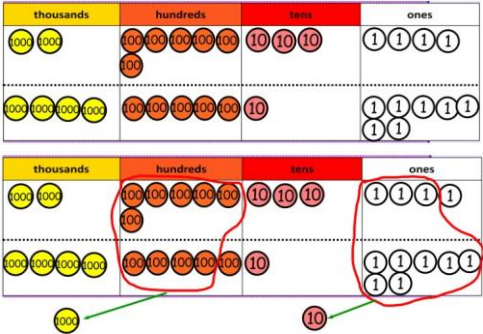
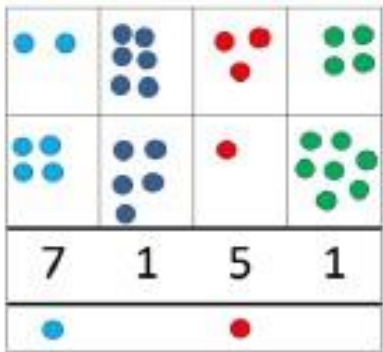
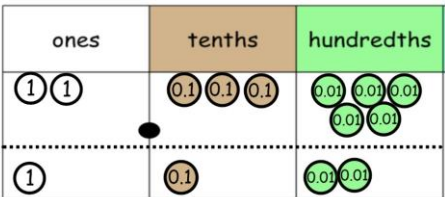
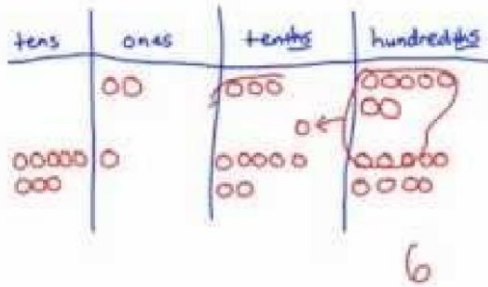


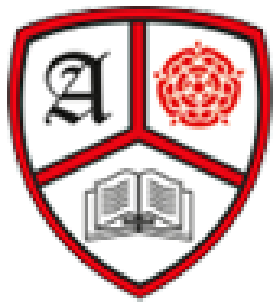
<b>Objective, Strategy</b> <b>Key Vocabulary</b>	<b>Concrete</b> When moving from concrete to pictorial, show concrete alongside pictorial. Show pictorial alongside abstract when moving to abstract.	<b>Pictorial</b>	<b>Abstract</b>
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers.</p> 	<p>Model using Dienes or numicon</p>  <p>Add together the ones first, then the tens.</p>   <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p> 	$\begin{array}{r} 248 \\ + 131 \\ \hline 379 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p> <p>Use language of 'take and make' to describe carrying</p>	 <p>Exchange ten ones for a ten. Model using numicon and pv counters.</p>  <p>Calculations</p> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	<p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p> 	<p>Use expanded method ONLY WHEN NEEDED</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$



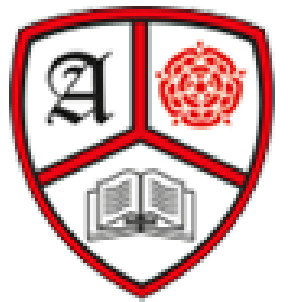
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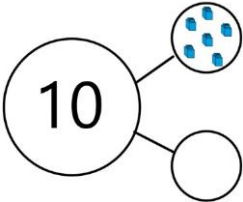
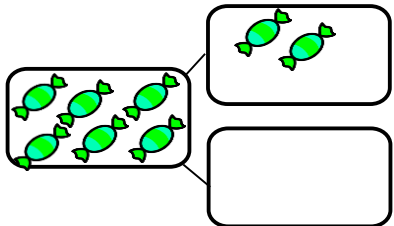
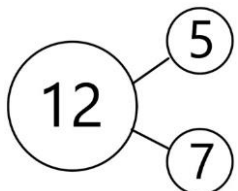
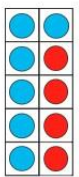
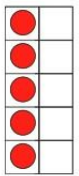
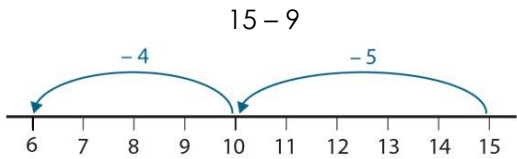
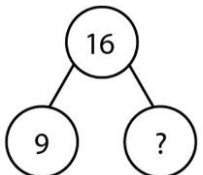
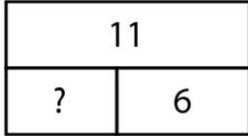
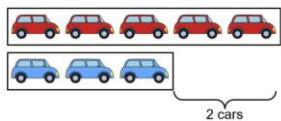
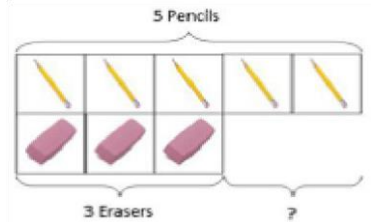
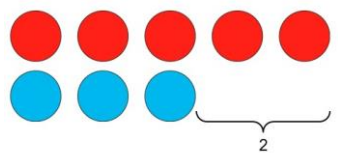
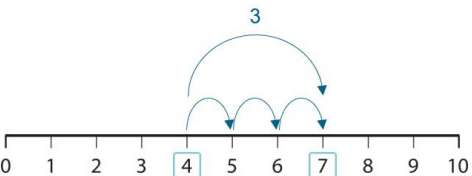


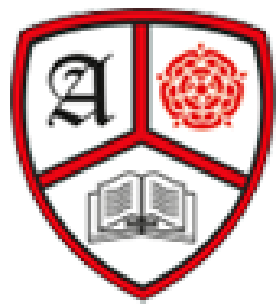
Objective ,Strategy Key Vocabulary	Concrete	Pictorial	Abstract
<p>Y4—add numbers with up to 4 digits</p>	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	$\begin{array}{r} 2634 \\ + 4517 \\ \hline 7141 \\ \hline \end{array}$ <p>Continue from previous work to carry ones, tens and hundreds. Relate to money and measures.</p>
<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters</p>		$\begin{array}{r} 22,634 \\ + 15,673 \\ \hline 38,307 \\ \hline \end{array}$ <p>£ 127.67 + £ 38.45 £ 166.12</p>
<p>Y6—add several numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>Some children may need to use manipulatives and/or representations for longer. See year 5</p>		$\begin{array}{r} 89,472 \\ 63,673 \\ + 3,016 \\ \hline 156,161 \\ \hline \end{array}$ <p>Insert zeros for place holders.</p> $\begin{array}{r} 1.437 \\ 0.600 \\ + 3.020 \\ \hline 4.057 \\ \hline \end{array}$



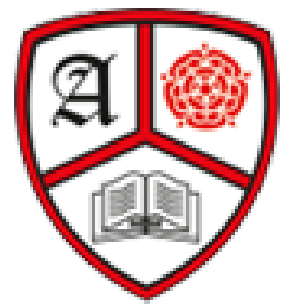
# Year 1 Subtraction

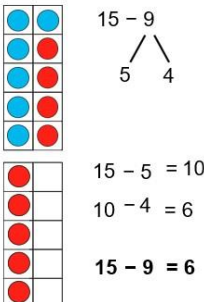
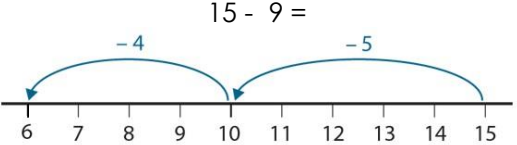
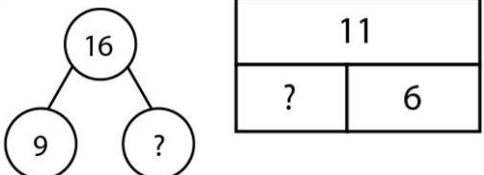
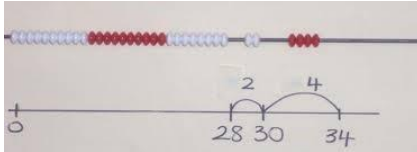
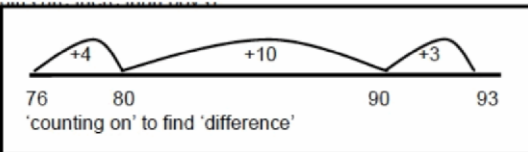
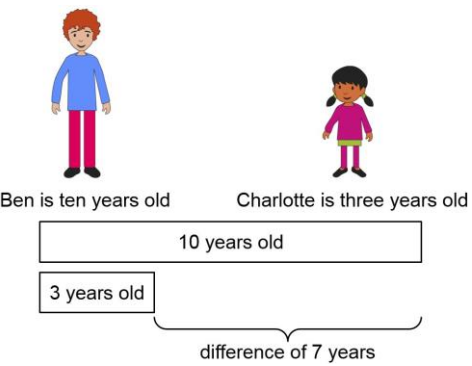
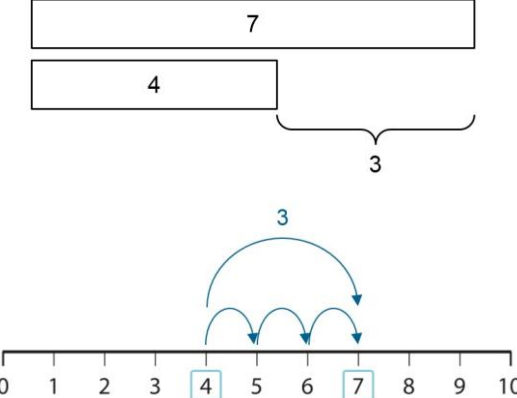


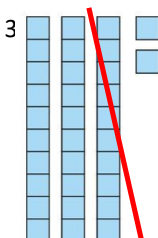
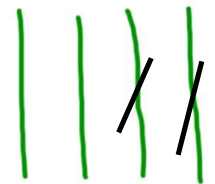
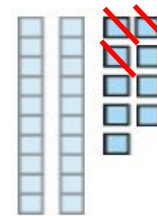
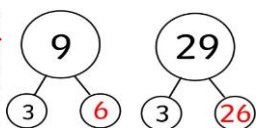
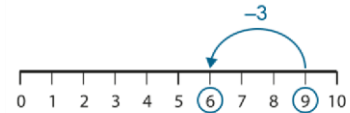
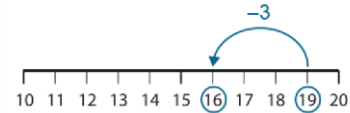
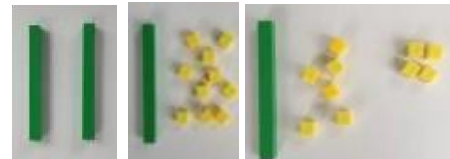
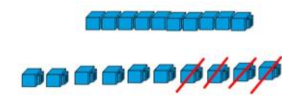
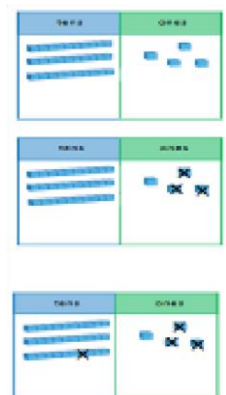
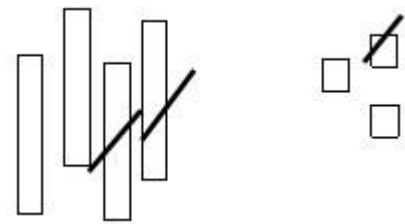
Objective, Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part-Part-Whole model	 <p>Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what's the other part? <math>10 - 6 = 4</math></p>	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p>  $12 - 5 = 7$ $12 - 7 = 5$ $7 = 12 - 5$ $5 = 12 - 7$
Subtract by making ten	<p><math>15 - 9</math> Make 15 on the ten frame. Take 5 away to make ten, then take 4 more away so that you have taken 9.</p>   $15 - 9 = 6$ $15 - 5 = 10$ $10 - 4 = 6$	<p><math>15 - 9</math></p>  <p>Jump back 5 first, then another 4. Use ten as the stopping point.</p>	<p><math>16 - 9</math> How many do we take off first to get to 10? How many left to take off?</p>  
Compare numbers by finding the difference.	 <p>There are 2 more red cars than blue cars.</p>  <p>There are 2 more pencils than erasers.</p>	 <p><math>5 - 3 = 2</math></p> <p>Use a number line to count on..</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>

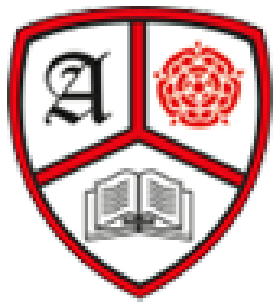


# Year 2 Subtraction

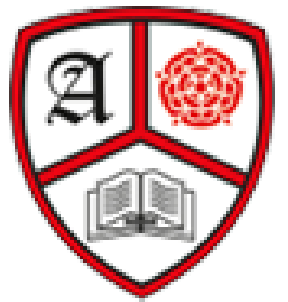


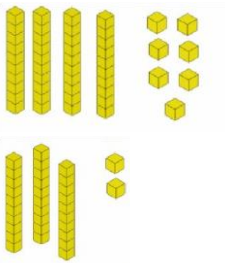
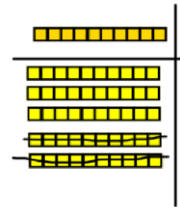
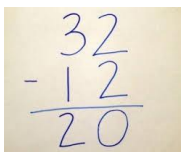
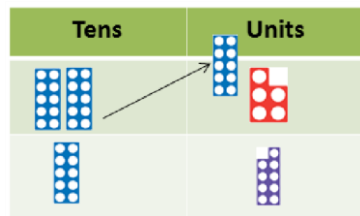
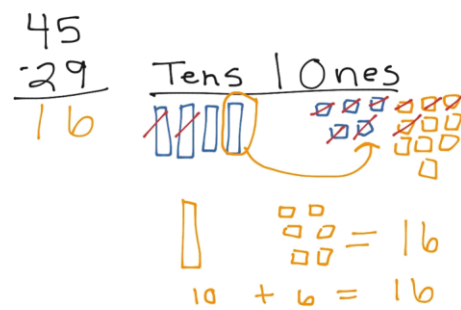
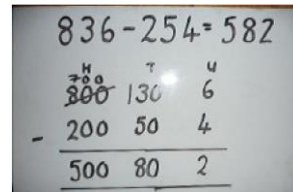
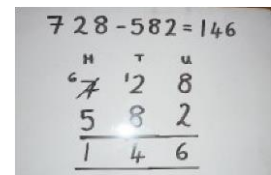
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting by making 10	<p><b>15 - 9 =</b></p> <p>Make 15 on the ten frame. Take 5 away to make ten, then take 4 more away so that you have taken 9.</p>  <p>15 - 9 =</p> <p>5      4</p> <p>15 - 5 = 10</p> <p>10 - 4 = 6</p> <p>15 - 9 = 6</p>	<p>15 - 9 =</p>  <p>Jump back 5 first, then another 4. Use ten as the stopping point.</p>	<p>16 - 9 =</p> <p>How many do we take off first to get to 10? How many left to take off?</p> 
<p>Counting on to next ten</p> <p><i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i></p>	<p>34 - 28 =</p>  <p>34 - 28</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p> <p>28 to 30 is 2, 30 to 34 is 4. So, 34 - 28 = 6</p>	 <p>Use a number line to count on to next ten and then the rest.</p> <p>Begin with bead line, move to landmarked line then to ENL.</p>	<p>93 - 76 = 17</p> <p>76 → 80 = 4</p> <p>80 → 93 = 13</p> <p>13 + 4 = 17</p>
Subtractions as difference	 <p>Ben is ten years old      Charlotte is three years old</p> <p>10 years old</p> <p>3 years old</p> <p>difference of 7 years</p>		<p>The difference between 24 and 16 is 8.</p>

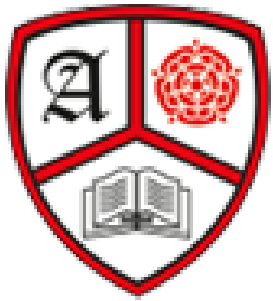
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting a multiple of 10	 $32 - 10 = 22$ Children use dienes, PV counters or Numicon. They remove the correct number of tens	 Children draw rods and cubes and cross off multiples of ten.	$64 - 10 = \square$ $64 - 20 = \square$ $64 - 30 = \square$ $64 - \square = 24$ $\square - 50 = 14$
Subtract a single digit from a two digit number No regrouping	 	 $9 - 3 = 6$  $19 - 3 = 16$	$9 - 3 = 6$ $19 - 6 = 13$ $29 - 6 = 23$ etc
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'.	$20 - 4 = 16$ 	$20 - 4 = 16$
Partitioning to subtract without regrouping. 'Friendly numbers'	$34 - 13 = 21$ Use Dienes to show how to partition the number when subtracting without regrouping. 	$43 - 21 = 22$ Children draw representations of Dienes and cross off. 	$43 - 21 = 22$



# Year 3 Subtraction

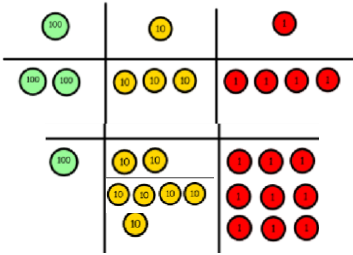
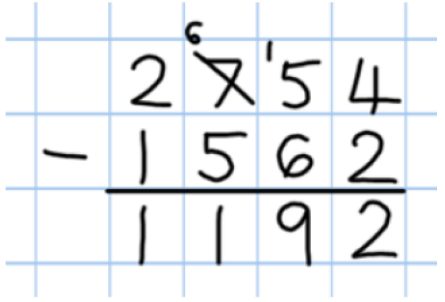
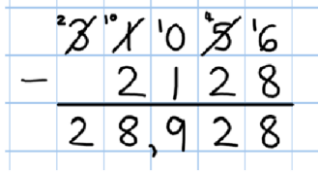
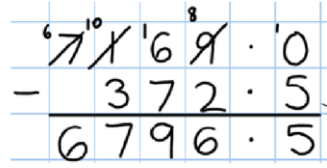
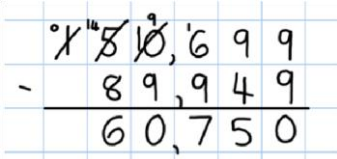
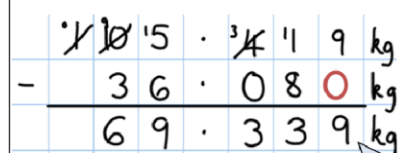


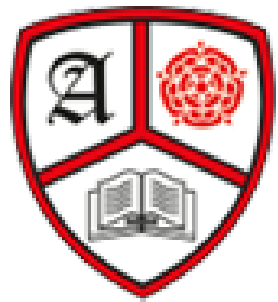
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping (friendly numbers)</p>	 $47 - 32 = 15$ <p>Use base 10 or Numicon to model</p>	 <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>



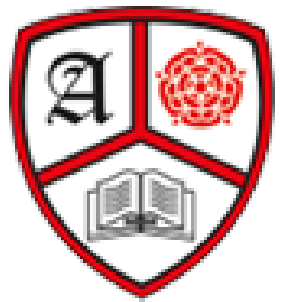
# Year 4 – 6 Subtraction

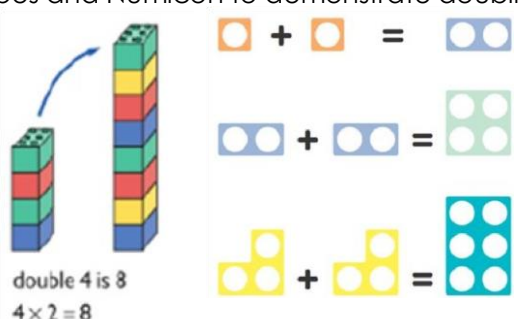
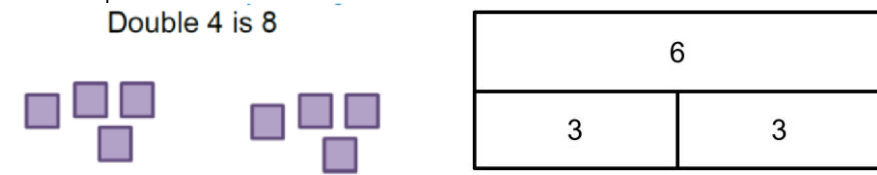

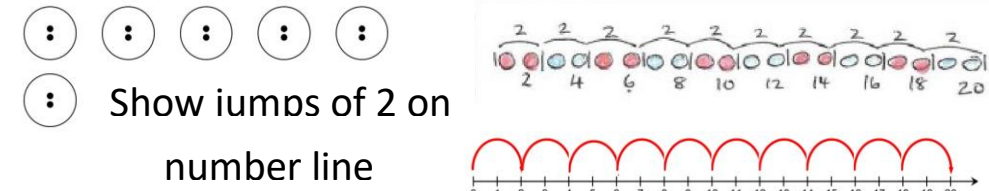
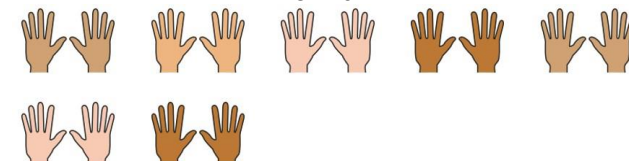
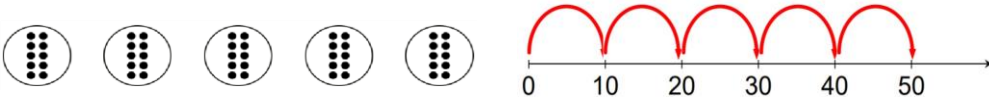

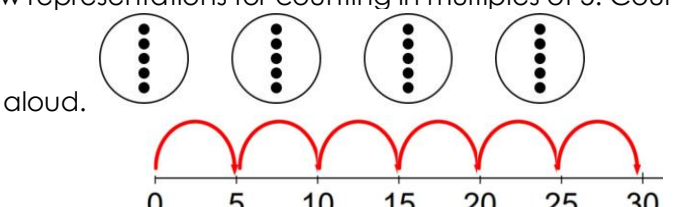


Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal point.</i></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for placeholders.</p> 
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</p>			 



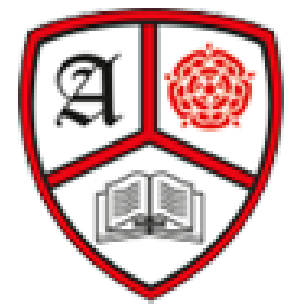
# Year 1 Multiplication

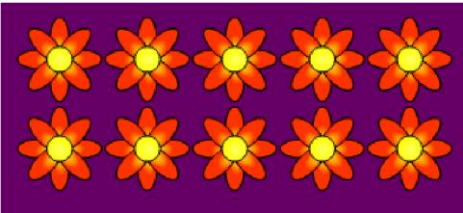
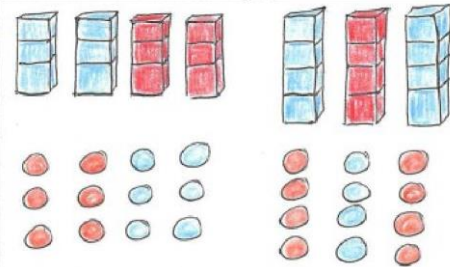
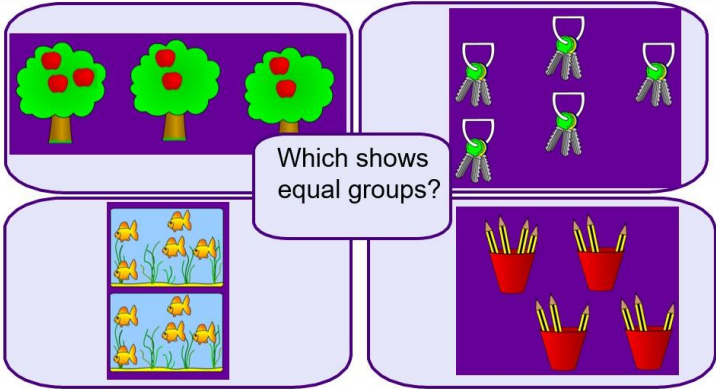
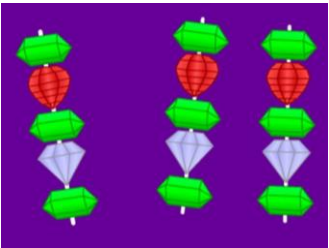
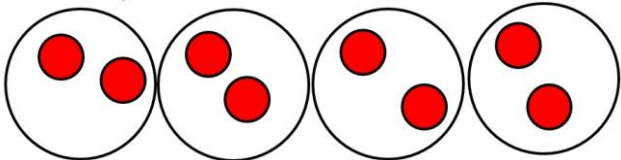


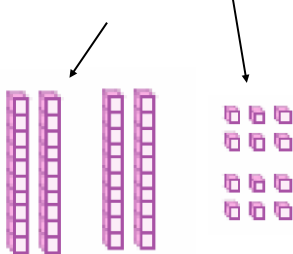

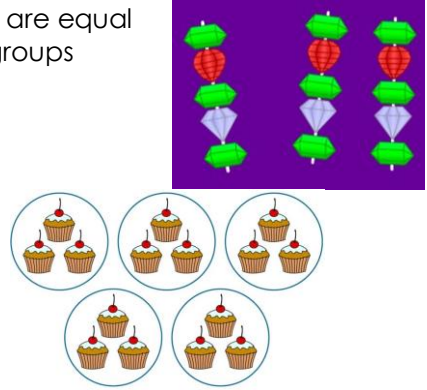
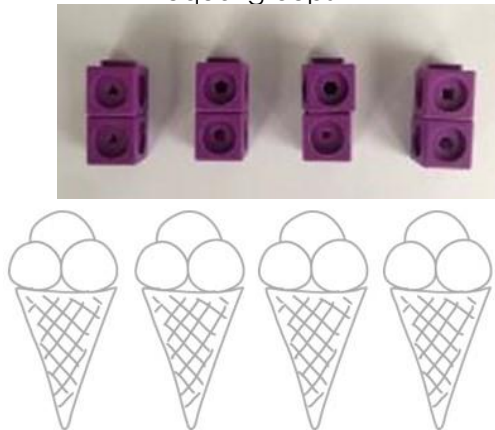
Objective & Strategy	Concrete	Pictorial
Double numbers to 10	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures and bar models to show how to double numbers</p> <p>Double 4 is 8</p> 
Counting in groups of 2	<p>Count in 2s using real life objects and contexts.</p> 	<p>Children make representations to show counting in multiples of 2. Count in multiples of a number aloud.</p>  <p>Show jumps of 2 on number line</p>
Counting in groups of 10	<p>Use real life objects and contexts to count in groups of 10</p> 	<p>Use and draw representations for counting in multiples of 10. Count in multiples of 10 aloud</p> <p>Show jumps of 10 on a number line</p> 
Counting in groups of 5	<p>Use real life objects and contexts to count in groups of 5</p> 	<p>Use and draw representations for counting in multiples of 5. Count in 5s aloud.</p> 


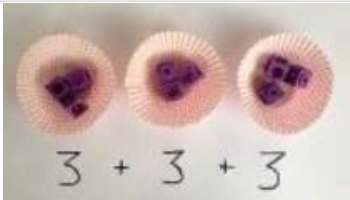
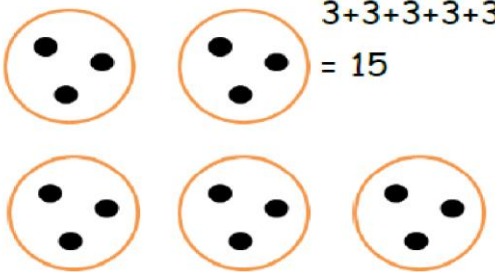
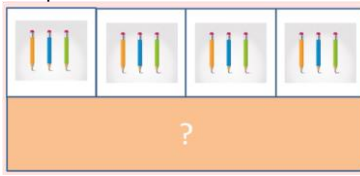
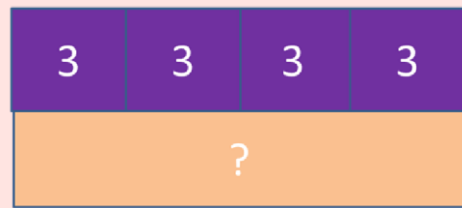
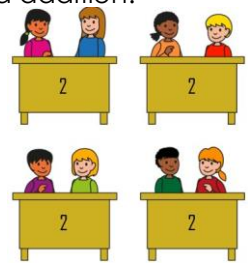




# Year 2 Multiplication



Objective & Strategy	Concrete	Pictorial
<p>Understand and use arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc.</p> 	<p>Make and draw representations of arrays to show understanding</p> 
<p>Equal/non equal groups</p>	<p>Use real life objects and contexts to examine equal and non-equal groups.</p> <div data-bbox="504 708 1218 1098">  <p>Which shows equal groups?</p> </div> <div data-bbox="555 1118 882 1366">  </div> <p>There are 3 equal groups. There are 5 in each group.</p>	<p>Children make/match representations of real life problems to show equal groups and find the total.</p>  <p>There are 4 equal groups. There are 2 in each group. There are 8 altogether.</p>

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Double a 2-digit number</p>	<p>Model doubling using dienes and PV counters.</p> $40 + 12 = 52$ 	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p> $  \begin{array}{r}  16 \\  \swarrow \quad \searrow \\  10 \quad 6 \\  \downarrow \times 2 \quad \downarrow \times 2 \\  20 \quad 12 \\  + \quad + \\  32  \end{array}  $
<p>Understand equal and non-equal groups</p>	<p>These are non-equal groups</p>  <p>These are equal groups</p>  <p>There are 5 equal groups. Each group has 3 cakes.</p>	<p>Make representations and drawings of equal groups</p>  <p>I have 4 groups of 3.</p>	

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Use repeated addition for multiplications</p>	<p>Use objects and real life contexts.</p>  $2 + 2 + 2 + 2 + 2 = 10$  $3 + 3 + 3$ <p>There are 3 groups of 3. There are 9 altogether.</p>	<p>Make and draw representations to show repeated addition</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p>  $3 + 3 + 3 + 3 + 3 = 15$ <p>Use bar models for representations of repeated additions.</p> 	<p>Create number sentences using repeated addition to match representations.</p>  $3 + 3 + 3 + 3 = 12$
<p>Relate repeated addition to multiplication using the x sign.</p>	<p>Write multiplication sentences to match repeated addition.</p>  $2 + 2 + 2 + 2$ $4 \times 2$	<p>Children make and draw representations and record both an addition sentence and a multiplication sentence.</p>  $1 + 1 + 1 + 1 + 1 + 1 = 6$  $6 \times 1 = 6$ $\square \times \square = 8$	<p>Write multiplication sentences to match repeated addition, without the support of representations.</p> $2 + 2 + 2 + 2 + 2 = 10$ $5 \times 2 = 10$

## Objective & Strategy

## Concrete

## Pictorial

## Abstract

Understand the 2, 5 and 10 times table

Use objects and real life contexts for multiples of 2, 5 and 10

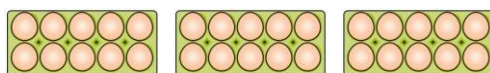
6



2 2 2

$$3 \times 2 = 6$$

$$6 = 3 \times 2$$



10

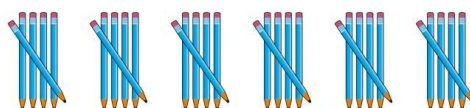
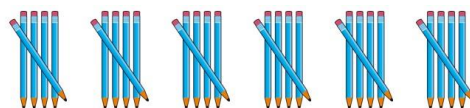
10  
ten

10

20  
twenty

30  
thirty

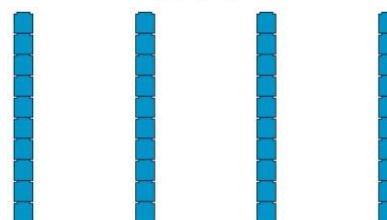
$$3 \times 10 = 30$$



Make and draw representations for 2, 5 and 10 times tables



$$12 = 6 \times 2$$



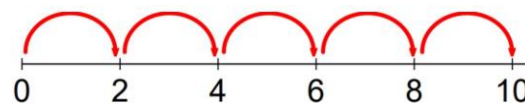
10 ten 20 twenty 30 thirty 40 forty

$$4 \times 10 = 40$$

Number lines, bead strings, counting sticks and bar models should be used

to show representation of counting in

multiples.



$$5 \times 2 = 10$$

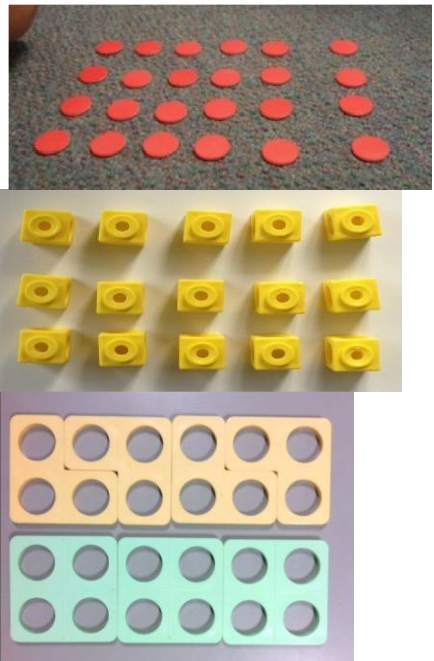
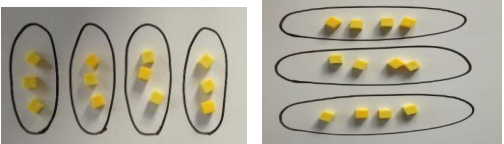
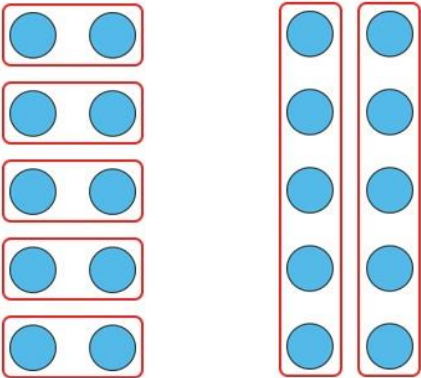
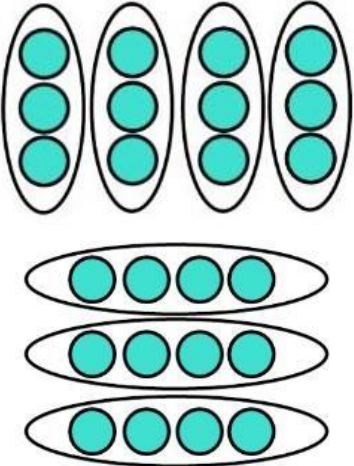



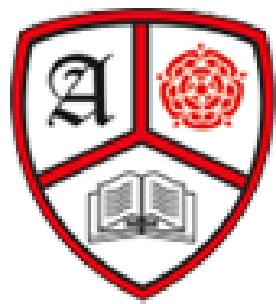
Understand the terms factor and product

3	×	2	=	6
factor	×	factor	=	product

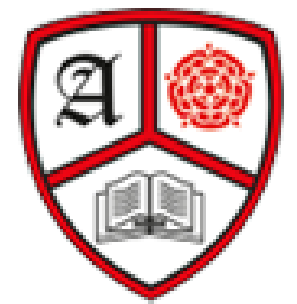
6	=	3	×	2
product	=	factor	×	factor

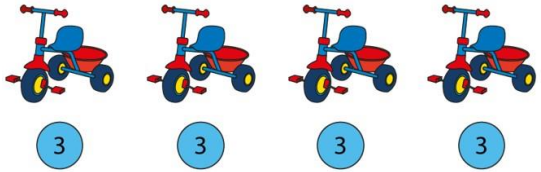
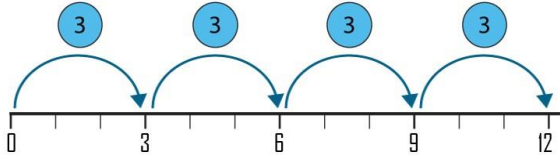
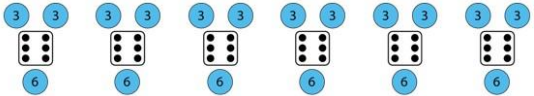
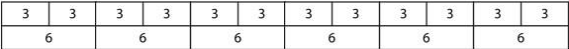
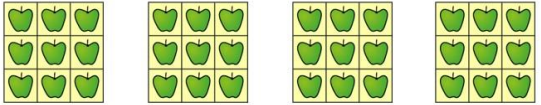
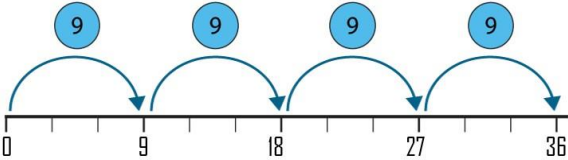
Count in multiples of a number aloud.

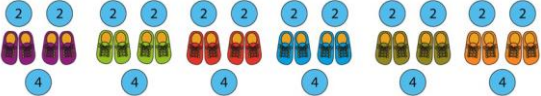
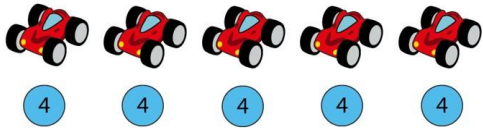
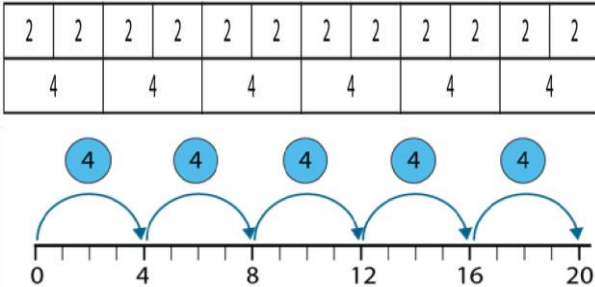
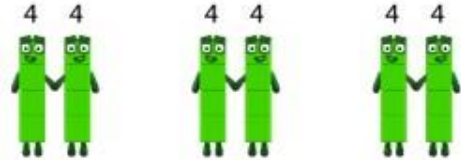


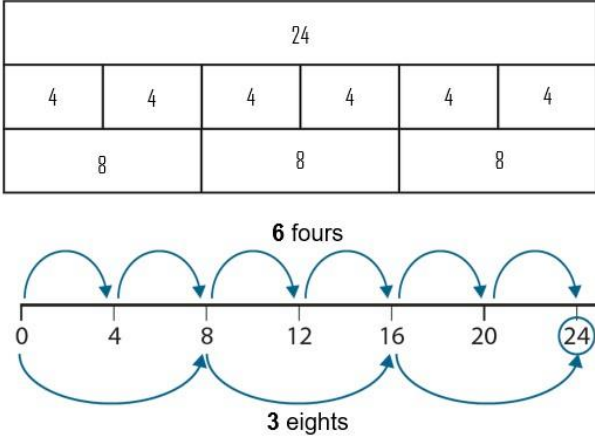
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  <p> <math>5 \times 2 = 10</math>      <math>5 \times 2 = 10</math>  5 groups of 2      2 groups of 5  2, five times      5, two times </p> 	<p><math>12 = 3 \times 4</math>   <math>12 = 4 \times 3</math></p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p> <math>5 + 5 + 5 = 15</math>  <math>3 + 3 + 3 + 3 + 3 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math> </p>



# Year 3 Multiplication



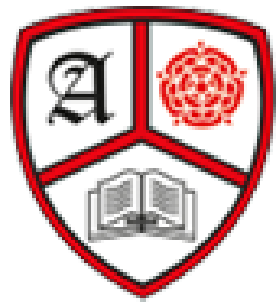
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Understand the 3 times table</p>	<p>Count in three using objects and representations of multiples of 3.</p> 		<p>There are 12 wheels.</p> $4 \times 3 = 12$ $3 \times 4 = 12$
<p>Understand the 6 times table</p>	<p>We can double our 3 times table to find our 6 times table.</p> 		$12 \times 3 = 36$ $6 \times 6 = 36$
<p>Understand the 9 times table</p>	<p>Count in nines using objects and representations of multiples of 9. Make links 9 being three groups of three.</p> 		<p>There are 36 apples.</p> $4 \times 9 = 36$ $9 \times 4 = 36$

Objective & Strategy	Concrete	Pictorial	Abstract
<p><i>Understand the 4 times table</i></p>	<p>We can double our 2 times table to get the 4 times table</p>  <p>Pupils revise 2 times table from year 2 and make link that this can be doubled to obtain 4 times table.</p> <p>How many wheels? Count in groups of 4.</p> 		$12 \times 2 = 24$ $6 \times 2 = 24$ <p><b>There are 20 wheels.</b></p> $5 \times 4 = 20$ $4 \times 5 = 20$
<p><i>Understand the 8 times table</i></p>	<p>We can double our 4 times table to get the 8 times table</p>   		$6 \times 4 = 24$ $3 \times 8 = 24$

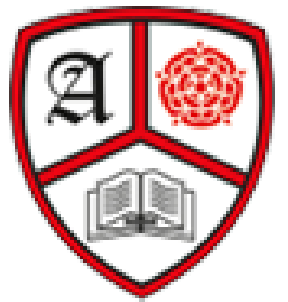
## Divisibility rules in 'families' – 2, 4 and 8

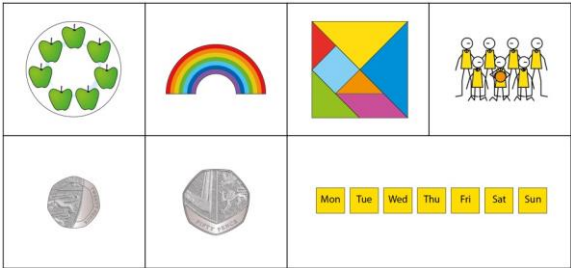
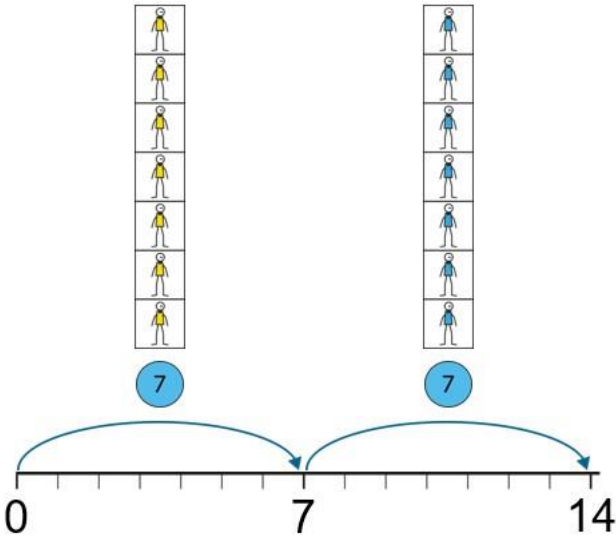
<b>2</b>	A number is divisible by 2 if the ones digit is even.
<b>4</b>	<p>If halving a number gives an even value, then the number is divisible by 4.</p> <p><i>and</i></p> <p>For numbers with more than two digits: if the final two digits are divisible by 4 then the number is divisible by 4.</p>
<b>8</b>	If halving a number twice gives an even value, the number is divisible by 8.



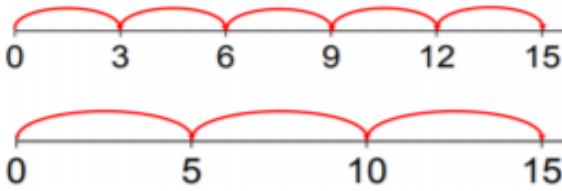


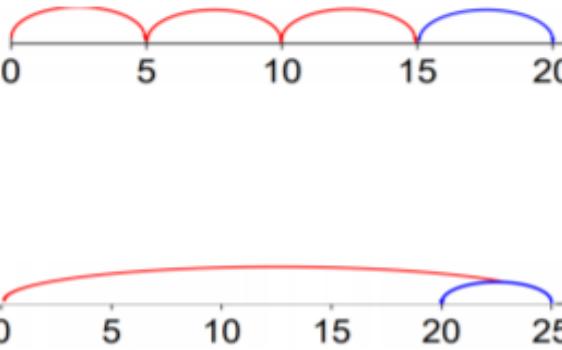
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplying 2-digit by 1 digit using partitioning (distributive law)</p>	<div data-bbox="465 180 779 300" data-label="Figure"> </div> <p>4 rows of 10 4 rows of 3</p> <p>Show the links with arrays to illustrate the PV partitioning</p> <p>Move onto base ten to move towards a more compact method.</p> <div data-bbox="465 523 739 659" data-label="Figure"> </div> <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> <div data-bbox="1025 411 1534 798" data-label="Figure"> </div>	$  \begin{aligned}  4 \times 10 &= 40 \\  4 \times 3 &= 12 \\  40 + 12 &= 52  \end{aligned}  $
<p>2 digit x 1 digit using PV counters (no regrouping)</p>	<p>23 x 3</p> <div data-bbox="510 946 902 1273" data-label="Figure"> </div> <p>Chn can see array in the ones and the tens. There is a visual link to repeated addition.</p>	<p>Children practice, drawing their representations.</p> <p>23 x 3</p> <div data-bbox="1238 1042 1496 1361" data-label="Figure"> </div>	$  \begin{array}{r}  23 \\  \times 3 \\  \hline  69  \end{array}  $


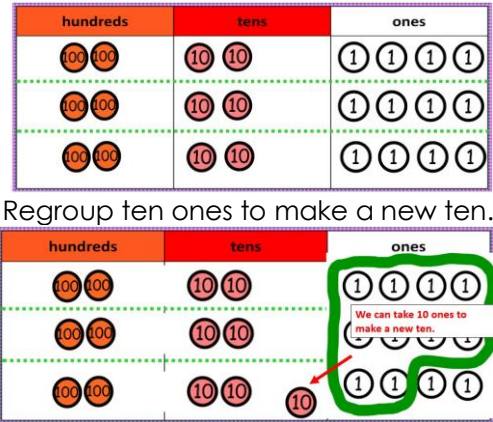
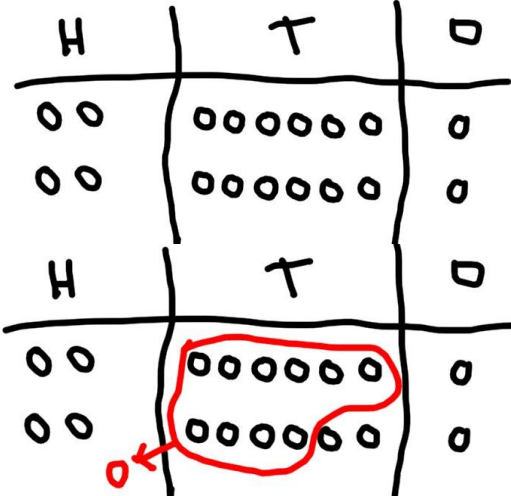


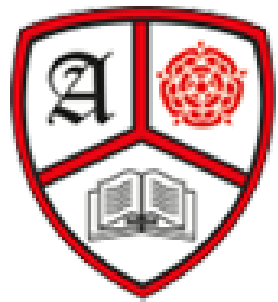
# Year 4 Multiplication



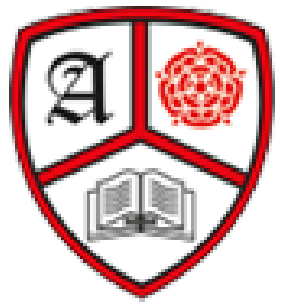
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Understand the 7 times table</p>	<p>Children use representations which show</p> <div data-bbox="392 708 960 976">  </div> <p>groups of 7 including real life contexts.</p>	<p>Linear models show jumps of 7.</p> <div data-bbox="1037 604 1650 1141">  </div>	<p>There are 14 players.</p> $2 \times 7 = 14$ $7 \times 2 = 14$
















Objective & Strategy	Concrete	Pictorial	Abstract
<p>Understanding the commutative law.</p>	<p>A</p>  <ul style="list-style-type: none"> <li>• 'Three groups of five are equal to fifteen.'</li> <li>• 'Five, three times is equal to fifteen.'</li> </ul> <p>B</p>  <ul style="list-style-type: none"> <li>• 'Five groups of three are equal to fifteen.'</li> </ul> <p>"Three groups of five is equal to five groups of three."</p>		$3 \times 5 = 15$ $5 \times 3 = 15$ $5 \times 3 = 3 \times 5 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$
<p>Understanding the distributive law</p>	 		$4 \times 5 = 3 \times 5 + 5 = 20$ $4 \times 5 = 5 \times 5 - 5 = 20$

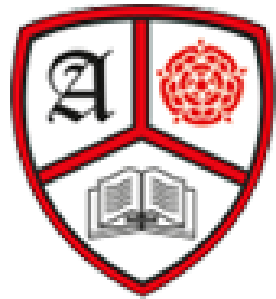
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiply 3 digit numbers by 1 digit. (no exchange)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 3 so we need 3 rows</p> $123 \times 3 = 369$  <p>300 + 60 + 9</p> <p>Add up each column, starting with the ones.</p>	<p>Children can represent their work with place value counters by drawing place value counters or Dienes.</p>	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <math display="block">\begin{array}{r} 231 \\ \times \quad 3 \\ \hline 693 \end{array}</math> </div> <div> <p>3 x 1 <i>ones</i> is three ones</p> <p>3 x 3 <i>tens</i> is nine tens</p> <p>3 x 2 <i>hundreds</i> is six hundreds</p> </div> </div>
<p>Multiply 3 digit numbers by 1 digit. (with exchange)</p>	<p>224 x 3</p>  <p>Regroup ten ones to make a new ten.</p> <p>600 + 70 + 2 = 672</p>	<p>261 x 2</p>  <p>500 + 20 + 2 = 522</p>	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <math display="block">\begin{array}{r} 241 \\ \times \quad 4 \\ \hline 964 \\ \textcolor{red}{1} \end{array}</math> </div> <div> <p>4 times 1 <i>ones</i> is 4 ones</p> <p>4 times 4 <i>tens</i> is 16 tens. I put 6 tens down and carry ten tens which is now a hundred.</p> <p>4 times 2 <i>hundreds</i> is 8 hundreds. I add the hundred I have carried to make 9 hundreds.</p> </div> </div>



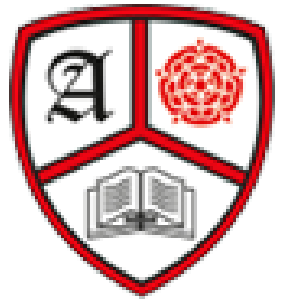
# Year 5 Multiplication



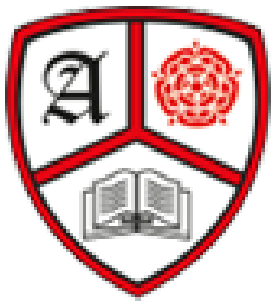
Objective & Strategy	Concrete	Pictorial	Abstract												
Multiply 3 and 4 digits x 1 digit.	<p>Children may continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping.</p> <p>3024 x 3</p> <table border="1"><thead><tr><th>thousands</th><th>hundreds</th><th>tens</th><th>ones</th></tr></thead><tbody><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></tbody></table> <p>900 + 0 + 60 +</p>	thousands	hundreds	tens	ones									Children may continue to draw their understanding using place value grids.	<div><div>3024</div><div>x3</div><div>9072</div><div>1</div></div>
thousands	hundreds	tens	ones												
															
															
Multiply up to 4 digits by 2 digits	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p> <p>Begin with teen number x teen number.</p> <p>Progress to any 2 –4 digit number x 2 digit.</p>	<div><div><div>108</div><div>10080</div><div>3024</div></div><div>→</div><div><div>183</div><div>54</div><div>180</div><div>234</div></div></div>	<div><div>18x3 on the first row</div><div>(8 x 3 =24, carrying the 2 for 20, then 1 x 3)</div><div>18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</div></div> <div><div><div>100s10s1s</div><div>31</div><div>24</div><div>124</div><div>620</div><div>744</div></div><div><div>31x4</div><div>31x20</div></div></div>												



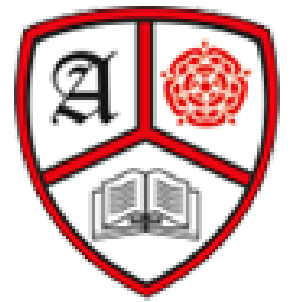
# Year 6 Multiplication

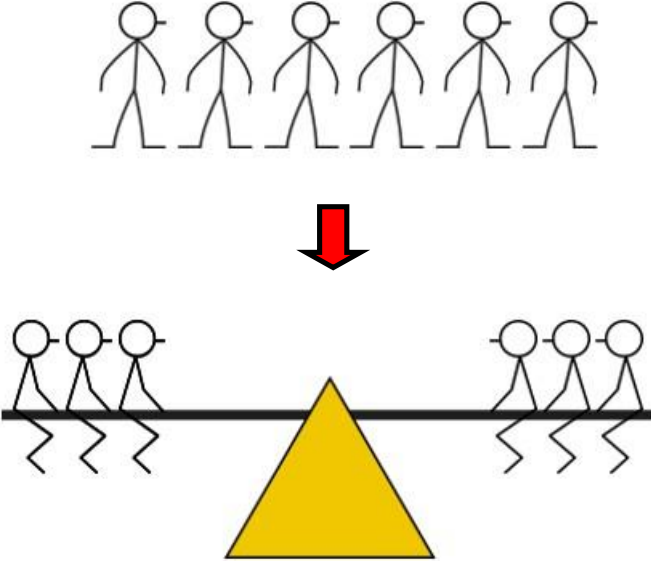
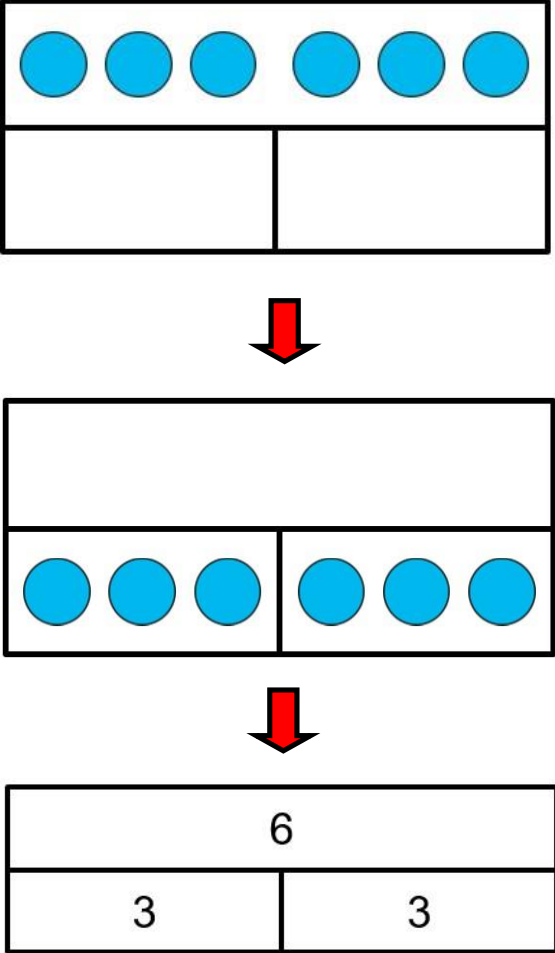


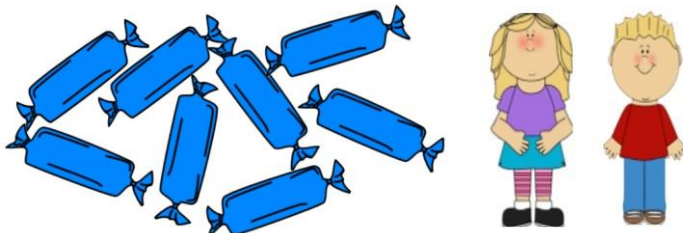
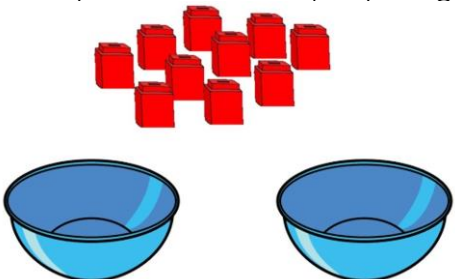
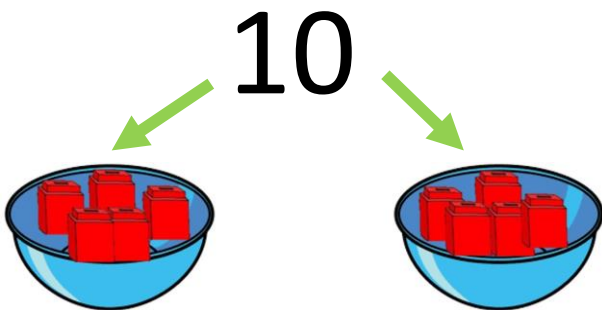
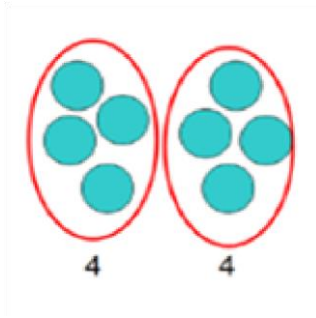
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiply decimals up to 2 decimal places by a single digit</p>			$  \begin{array}{r}  2.38 \\  \times \quad 3 \\  \hline  714 \\  \textcolor{red}{1} \text{ } 2  \end{array}  $ <p>First we lay out the calculation</p> <p>Next, we write the decimal point in the answer (product).</p> <p>Finally, we carry out the multiplication.</p> <p><i>3 x 8 hundredths is 24 hundredths</i></p> <p><i>3 x 3 tenths is 9 tenths, add 2 tenths we carried is 11 tenths</i></p> <p><i>3 x 3 ones is 6 ones, add 1 one we carried is 7 ones</i></p>
<p>Multiply up to 4 digit numbers by 2 digits.</p>			$  \begin{array}{r}  \phantom{\times} \quad \quad \quad \cancel{4} \\  \phantom{\times} \quad \quad \quad 3 \phantom{0} 1 \phantom{0} 2 \\  \times \phantom{\phantom{\times} \quad \quad \quad} \phantom{\phantom{\times} \quad \quad \quad} 2 \phantom{0} 8 \\  \hline  2 \phantom{0} 4 \phantom{0} 9 \phantom{0} 6 \\  6 \phantom{0} 2 \phantom{0} 4 \phantom{0} 0 \\  \hline  8 \phantom{0} 7 \phantom{0} 3 \phantom{0} 6 \\  \hline  1  \end{array}  $

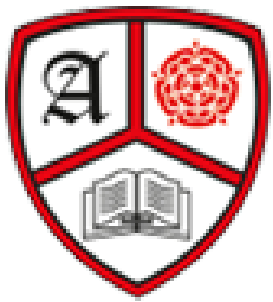


# Year 1 Division

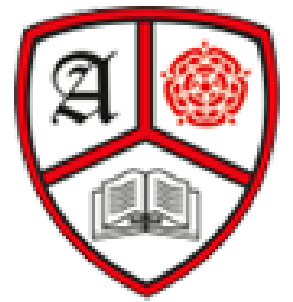


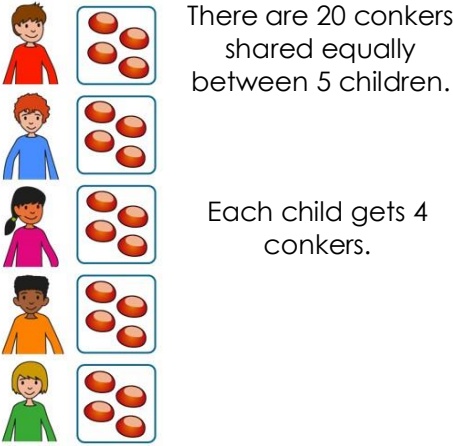
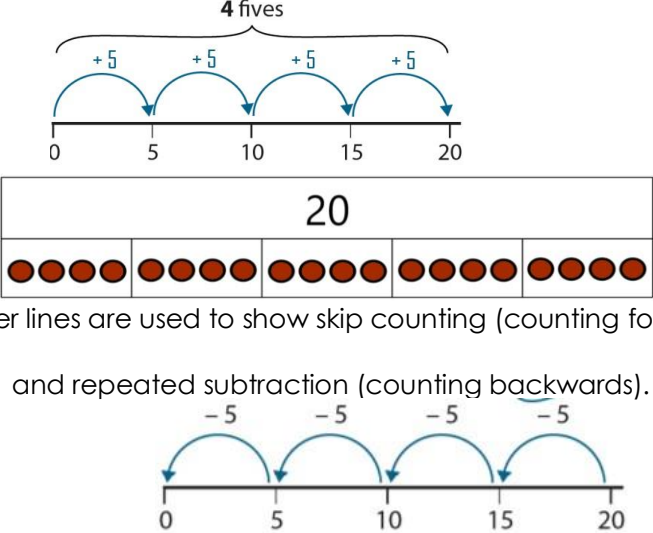
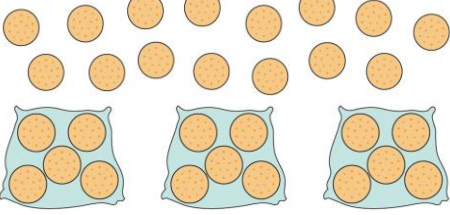
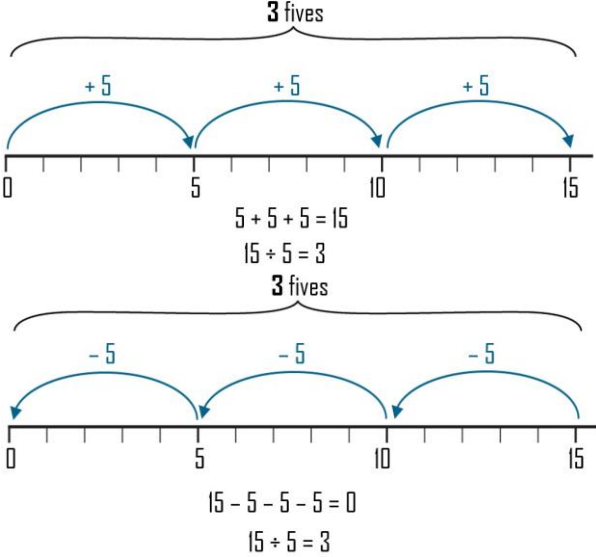
Objective & Strategy	Concrete	Pictorial
<p>Find half of numbers to 20.</p>	<p>Real life and practical contexts are used to find half of numbers up to 20.</p>  <p>The diagram shows a sequence of steps. First, six stick figures are shown in a row. A red arrow points down to a balance scale. On the left pan of the scale, three stick figures are placed, and on the right pan, three stick figures are placed. The scale is balanced on a yellow triangular fulcrum.</p>	<p>Children use manipulatives to represent real life problems.</p>  <p>The diagram shows a sequence of steps. First, a bar model is shown with six blue circles in the top row and two empty boxes in the bottom row. A red arrow points down to the same bar model, but now the top row is empty and the bottom row contains six blue circles, with three circles in each of the two boxes. A second red arrow points down to a final bar model where the top row contains the number 6 and the bottom row contains the number 3 in each of the two boxes.</p> <p>half of 6 = 3 double 3 = 6</p>

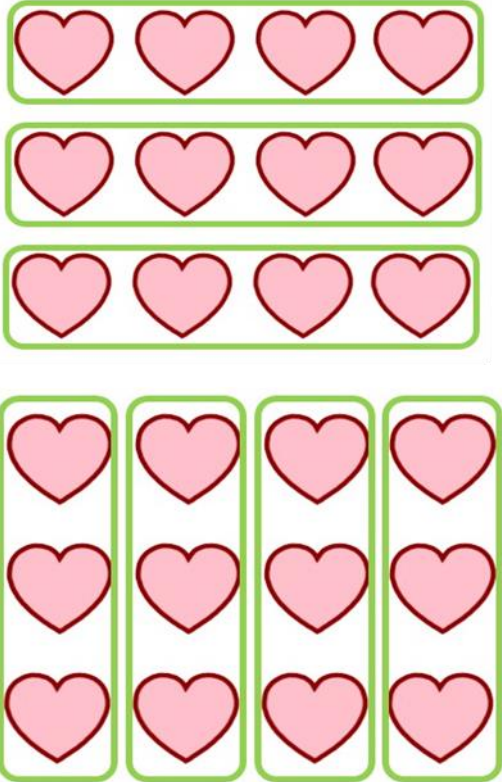
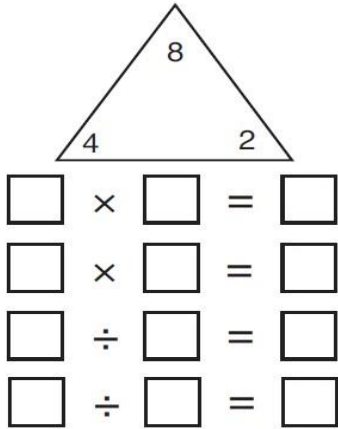
Objective & Strategy	Concrete	Pictorial
<p>Understand division as sharing into equal groups</p> <p>Use White Rose ITPs for modelling</p>	<div data-bbox="465 245 1146 478">  </div> <p data-bbox="504 486 1160 518">Children solve real life problems using real objects.</p> <p data-bbox="427 651 1240 718">There are eight sweets. Daisy and Will share these equally. How many do they get each?</p> <p data-bbox="459 750 1209 782">I have 10 cubes, can you share them equally in 2 groups?</p> <div data-bbox="667 782 1120 1061">  </div> <p data-bbox="546 1069 1120 1101">There are 2 equal groups. Each group has 5.</p> <div data-bbox="573 1133 1173 1444">  </div>	<p data-bbox="1370 619 2027 651">Children use pictures or shapes to share quantities.</p> <p data-bbox="1545 686 1854 718">8 shared between 2 is 4</p> <div data-bbox="1534 758 1848 1077">  </div>

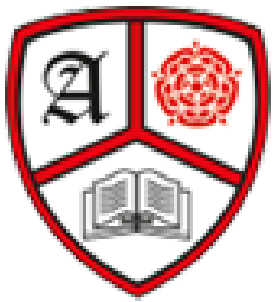


# Year 2 Division

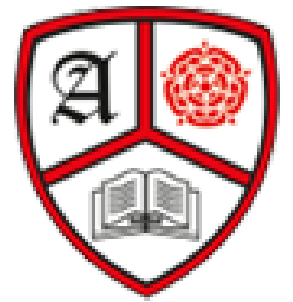


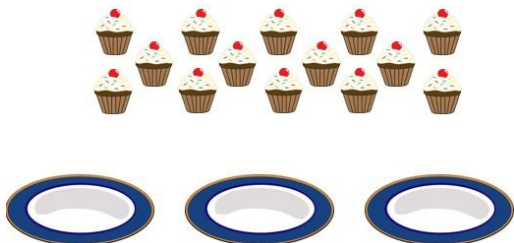
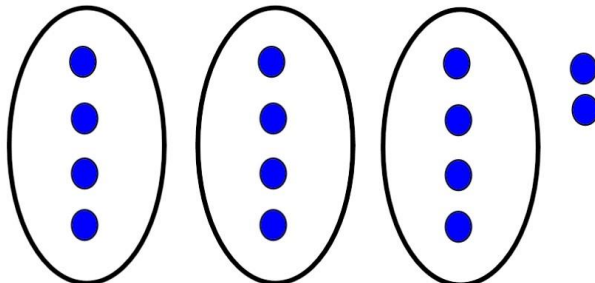


Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing (partitive)</p>	 <p>There are 20 conkers shared equally between 5 children.</p> <p>Each child gets 4 conkers.</p>	<p>Children use pictures or shapes to share quantities. They may use bar modelling to show and support understanding.</p>  <p>Number lines are used to show skip counting (counting forwards) and repeated subtraction (counting backwards).</p>	$20 \div 5 = 4$
<p>Division as grouping (quotitive)</p>	<p>Use cubes, counters or real objects or to aid understanding.</p> <p>There are 15 biscuits, there are 5 in each bag. How many bags?</p> 	 <p>15 divided into groups of 5 is 3</p>	$15 \div 5 = 3$

Objective & strategy	Concrete	Pictorial	Abstract
Understanding the inverse			$3 \times 4 = 12$ $12 \div 4 = 3$ $4 \times 3 = 12$ $12 \div 3 = 4$ $2 \times 4 = 8$ $8 \div 2 = 4$ $8 = 2 \times 4$ $2 = 8 \div 4$ $4 \times 2 = 8$ $8 \div 4 = 2$ $8 = 4 \times 2$ $4 = 8 \div 2$ <p>Show all 8 related fact family sentences.</p>



# Year 3 Division



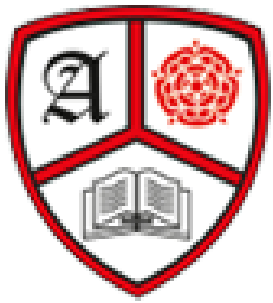
Objective & Strategy	Concrete	Pictorial	Abstract										
Division with remainders. (partitive)	<p>I divide 14 cakes between 3 plates. How are the cakes shared?</p> 	<p>Draw dots and group them to divide an amount and clearly show a remainder.</p> 	<p>Complete written divisions and show the remainder using r.</p> <div><div>14</div><div>÷</div><div>3</div><div>=</div><div>4</div><div>r</div><div>2</div></div> <div><div>↓</div><div>dividend</div></div> <div><div>↓</div><div>divisor</div></div> <div><div>↓</div><div>quotient</div></div> <div><div>↓</div><div>remainder</div></div>										
Division with remainders. (quotitive)	<p>13 eggs are put into boxes. Each box holds 3 eggs. How are the eggs boxed?</p> 	<p>Children may draw representations to show their understanding.</p>  <p>Use bar models to show division with remainders.</p> <table><tr><td colspan="5">13</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>1</td></tr></table>	13					3	3	3	3	1	<p>13 ÷ 3 = 4 r 1</p>
13													
3	3	3	3	1									

### **Divisibility rules in 'families' – 3, 6 and 9**

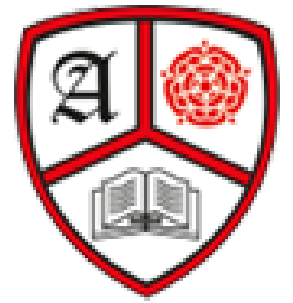
<b>3</b>	For a number to be divisible by 3, the sum of the digits of the number must be divisible by 3.
<b>6</b>	For a number to be divisible by 6, the number must be divisible by <i>both 2 and 3</i> .
<b>9</b>	For a number to be divisible by 9, the sum of the digits of the number must be divisible by 9.

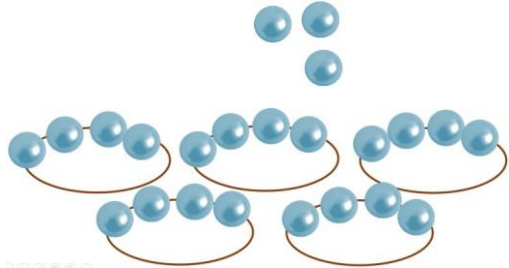
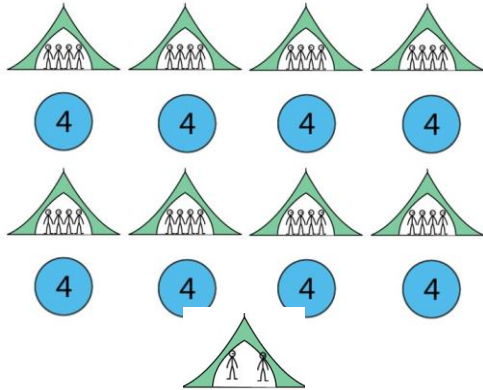
### **Divisibility rules in 'families' – 5 and 10**

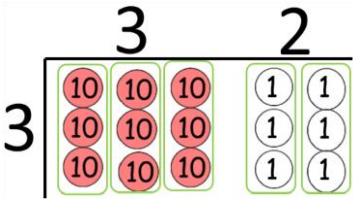
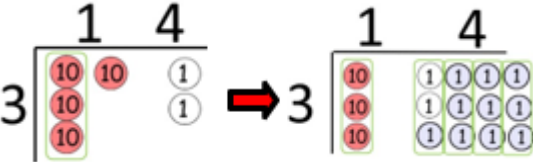
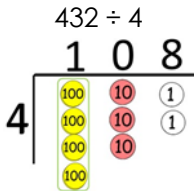
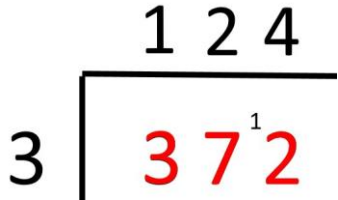
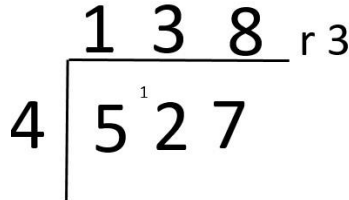
<b>5</b>	A number is divisible by 5 if the ones digit is 5 or 0.
<b>10</b>	A number is divisible by 10 if the ones digit is 0.



# Year 4 Division

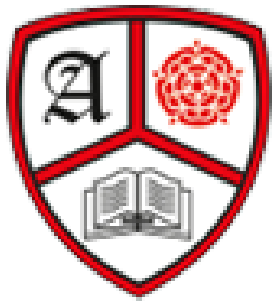


Objective & Strategy	Concrete	Pictorial	Abstract																
Interpreting division with remainders.	<p>Bracelets are made using 4 beads. There are 23 beads. How many bracelets can be made? How many beads left over?</p> 	<p>Bar model representations may be used.</p> <table border="1"><tr><td colspan="6">23</td></tr><tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>3</td></tr></table>	23						4	4	4	4	4	3	$23 \div 4 = 5 \text{ r } 3$				
23																			
4	4	4	4	4	3														
Interpreting division with remainders.	<p>4 scouts can fit in each tent. How many tents needed for 30 scouts?</p> 	<table border="1"><tr><td colspan="8">30</td></tr><tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>2</td></tr></table>	30								4	4	4	4	4	4	4	2	$30 \div 4 = 7 \text{ r } 2$ <p>8 tents are needed.</p> <p>Discuss with pupils the need to round up in this context.</p>
30																			
4	4	4	4	4	4	4	2												

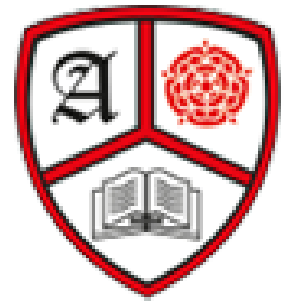
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Divide 2 &amp; 3 digit numbers by 1 digit</p> <p>Short Division</p>	<p><math>96 \div 3</math></p> <p>Use place value counters to make groups of the divisor, starting with the largest value digit.</p>  <p>There are 3 groups of 3 tens. There are 2 groups of 3 ones.</p> <p><math>42 \div 3</math></p>  <p>There is 1 group of 3 tens. There is a ten left over. We exchange this for 10 ones. 12 ones divided by 3 is 4.</p> <p><math>432 \div 4</math></p>  <p>There is 1 group of 4 hundreds. There are no groups of 4 tens and 3 tens left over. There are 8 groups of 4 ones after exchanging the left over tens.</p>	<p>Students use drawn diagrams with spots or circles to show their understanding.</p>	<p>Begin with divisions that divide equally with no remainder.</p>  <p>Move on to divisions with a remainder. Return to concrete if necessary.</p> 

Divisibility rules in numerical order	
<b>2</b>	A number is divisible by 2 if the ones digit is even.
<b>3</b>	For a number to be divisible by 3, the sum of the digits of the number must be divisible by 3.
<b>4</b>	<p>If halving a number gives an even value, then the number is divisible by 4.</p> <p><i>and</i></p> <p>For numbers with more than two digits: if the final two digits are divisible by 4 then the number is divisible by 4.</p>
<b>5</b>	A number is divisible by 5 if the ones digit is 5 or 0.

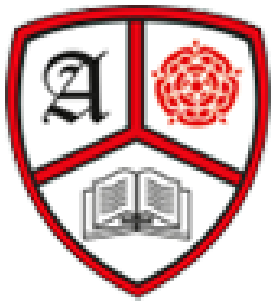
Divisibility rules in numerical order	
<b>6</b>	For a number to be divisible by 6, the number must be divisible by <i>both 2 and 3</i> .
<b>8</b>	If halving a number twice gives an even value, the number is divisible by 8.
<b>9</b>	For a number to be divisible by 9, the sum of the digits of the number must be divisible by 9.
<b>10</b>	A number is divisible by 10 if the ones digit is 0.



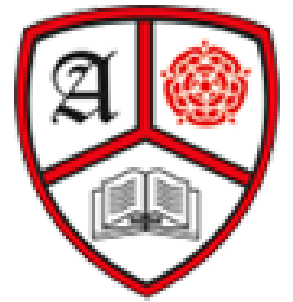
# Year 5 Division



Objective & Strategy	Concrete	Pictorial	Abstract
Divide decimals by a single digit, using x and ÷ by 10 or 100			<p>Pupils understand the use of X and ÷ 10 to make connections.</p> $  \begin{array}{rcl}  6.3 & \div & 9 = 0.7 \\  \times 10 \downarrow & & \uparrow \div 10 \\  63 & \div & 9 = 7  \end{array}  $
Short division of decimals			<p>Children build on work from Year 4, now with decimals.</p> $  \begin{array}{r}  0 \cdot 41 \\  6 \overline{) 2 \cdot 246}  \end{array}  $



# Year 6 Division



### Division of 2 digits by 2 digits

Using  $\times$  &  $\div$  by 10, 100 etc and relating this to a short division method.

$$\begin{array}{ccccc} \mathbf{60} & \div & \mathbf{30} & = & \boxed{2} \\ \div 10 \downarrow & & \downarrow 10 \div & & \uparrow \\ 6 & \div & 3 & = & 2 \end{array}$$

$$\begin{array}{r} 0 \quad 2 \\ 30 \overline{) 6 \quad 0} \end{array}$$

# Long Division—2 digits divided by 2 digits

T O

$$\begin{array}{r} 30 \overline{) 85} \end{array}$$

30 does not go into 8.  
So, combine the 8  
tens with the 5 ones.

T O

$$\begin{array}{r} 2 \\ 30 \overline{) 85} \\ 60 \end{array}$$

30 goes into 85 twice,  
which is 60.

T O

$$\begin{array}{r} 2 \\ 30 \overline{) 85} \\ 60 \\ \hline 25 \end{array}$$

Subtract the 60 from  
the 85 and this leaves  
25.

T O

$$\begin{array}{r} 2 \text{ r } 25 \\ 30 \overline{) 85} \\ 60 \\ \hline 25 \end{array}$$

85 divided by 30 is 2  
with a remainder of  
25

## Long Division—3 digits divided by 2 digits

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 31 \overline{) 4 \quad 3 \quad 4} \end{array}$$



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \\ 31 \overline{) 4 \quad 3 \quad 4} \end{array}$$

31 does not go into 4 (hundreds).



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \quad 1 \\ 31 \overline{) 4 \quad 3 \quad 4} \\ \underline{3 \quad 1} \end{array}$$

We combine the 4 hundreds with the tens to give 43 tens. 31 goes into 43 once which is 31, we record this underneath.



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \quad 1 \\ 31 \overline{) 4 \quad 3 \quad 4} \\ \underline{3 \quad 1} \\ 1 \quad 2 \end{array}$$

Subtract to find the remainder. 31 from 43 leaves 12.

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \quad 1 \\ 31 \overline{) 4 \quad 3 \quad 4} \\ \underline{3 \quad 1} \\ 1 \quad 2 \quad 4 \end{array}$$

We combine 12 with the next digit to give 124.



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \quad 1 \\ 31 \overline{) 4 \quad 3 \quad 4} \\ \underline{3 \quad 1} \\ 1 \quad 2 \quad 4 \\ \underline{1 \quad 2 \quad 4} \end{array}$$

31 goes into 124 four times, which is 124.



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 0 \quad 1 \quad 4 \\ 31 \overline{) 4 \quad 3 \quad 4} \\ \underline{3 \quad 1} \\ 1 \quad 2 \quad 4 \\ \underline{1 \quad 2 \quad 4} \\ 0 \end{array}$$

We subtract to show there is no remainder

## Long Division—progressing to 4 or more digits

	TH	H	T	O	
23	)	4	9	4	5



	TH	H	T	O	
		2			
23	)	4	9	4	5
		4	6		
			3		



	TH	H	T	O	
		2	1		
23	)	4	9	4	5
		4	6		
			3	4	
			2	3	
			1	1	

23 goes into 49 twice which is 46. We subtract this from 49 to give a remainder of 3.

We combine the 3 left over with the next digit to give 34. 23 goes into 34 once with 11 remaining.



	TH	H	T	O	
		2	1	5	
23	)	4	9	4	5
		4	6		
			3	4	
			2	3	
			1	1	5
			1	1	5
					0

We combine the 11 with the next digit to make 115. 23 goes into 115 5 times with no remainder.

## Long Division—procedural summary (remainder in the tens)

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \end{array}$ <p>Two goes into 5 two times, or 5 tens <math>\div 2 = 2</math> whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ - 4 \phantom{0} \\ \hline 1 \phantom{0} \end{array}$ <p>To find it, multiply <math>2 \times 2 = 4</math>, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \phantom{0} \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \phantom{0} \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \phantom{0} \\ \hline 18 \\ - 18 \\ \hline 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ - 4 \phantom{0} \\ \hline 18 \\ - 18 \\ \hline 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

## Long Division—procedural summary (remainder in any of the digits)

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds <math>\div 2 = 1</math> hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply <math>1 \times 2 = 2</math>, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \downarrow \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply <math>3 \times 2 = 6</math>, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>